Estimation of the minimum integration time for determining the equivalent continuous sound level with a given level of uncertainty considering some statistical hypotheses for road traffic

S. R. de Donato\textsuperscript{a})

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In this work a relation is obtained for calculating the minimum time necessary for measuring the hourly equivalent level, with preset uncertainty on the $L_{eq}$ in the case of noise produced by road traffic under different statistical hypotheses for vehicular flow. A simple equivalent level prediction model is used as reference. Some specific relations between acoustic power and vehicle speed are implemented in this model. Through the application of the classic theory of errors, the expression for the uncertainty on the $L_{eq}$ is obtained with reference, in particular, to various vehicle distributions: uniform (rectangular), triangular, normal and Poisson that, according to the available information, can be applied for describing traffic flow. Uncertainties over the distance source/receiver and speed of the vehicles are also taken into consideration in the calculation of uncertainty on $L_{eq}$. The minimum measurement time is obtained from the expression of the error associated with the $L_{eq}$ according to the hourly number of vehicles, so that the uncertainty on the $L_{eq}$ stays within a preset value. In this case too, the determination of the minimum time refers to the various previously mentioned hypotheses with respect to vehicle distribution. It is shown that it is possible to obtain a correct description of road traffic noise, within a predetermined uncertainty on the hourly $L_{eq}$, by measuring over times considerably shorter than an hour. © 2007 Institute of Noise Control Engineering.

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1 INTRODUCTION

Definition of the uncertainty associated with environmental noise levels is currently the subject of particular attention since requests for assessing noise in urban areas\textsuperscript{1–4} are on the increase. The need to contain uncertainty on the measured value must however be in harmony with the necessity to reduce the time and resources to be used at the individual site, also in order to increase the number of measurement points and consequently improve the significance of the spatial analysis. To that end, use can be made of temporal sampling techniques through which the long-term value of environmental noise can be estimated starting from measurements on shorter time intervals\textsuperscript{5–7}. Since road traffic represents the greatest source of noise in urban areas, a real opportunity to optimise resources is given by the development of relations that will allow shortening of sampling times for determining the hourly equivalent level produced by road traffic with a predetermined uncertainty on the $L_{eq}$. In this work, starting from a simple model for predicting the $L_{eq}$ produced by vehicular traffic, a relation is obtained for calculating the uncertainty on the hourly $L_{eq}$ and from this is obtained the minimum integration time that guarantees a predetermined uncertainty on the $L_{eq}$ itself. The relation obtained allows measurements to be made over times that are considerably shorter than an hour, according to the hypotheses on traffic flow distribution. As regards vehicular flow some statistical distributions are taken into consideration. If no hypothesis can be formulated with regard to the distribution of vehicles, a uniform (rectangular) distribution is indicated; alternatively triangular, normal or Poisson distributions can be considered.

The uncertainty on the average power level of the vehicles is estimated considering traffic flows made up of different percentages of light and heavy vehicles, while for the variables distance of the traffic line from

\textsuperscript{a}) Regional Agency for Environmental Protection (ARPA), Via Gambalunga 83, 47900 Rimini, Italy. Electronic mail: sdedonato@arpa.emr.it.
the receiver and speed of the vehicles, uniform distributions are assumed for greater generality.

2 DESCRIPTION OF THE MODEL

With reference to Fig. 1, a vehicle moves along a straight road at an average speed \( v \) (m/s). The observation point \( R \) is at a distance \( d \) (m) from the road and at a distance \( r(t) \) (t) from point \( S \). Considering a single vehicle as a source point, the ground as a perfectly reflecting surface and ignoring the attenuation caused by air absorption, the expression of the \( L_{eq} \) can be obtained by integrating the sound energy produced by the passage of the source during time interval \( 2T \). Considering the symmetry of the situation:

\[
L_{eq} = 10 \log \left( \frac{1}{T} \int_{0}^{T} \frac{p^2(t)}{P_0^2} \, dt \right) = 10 \log \left( \frac{1}{T} \int_{0}^{T} \frac{2WS_0}{W_0^4 \pi^2 r^2(t)} \, dt \right) \tag{1}
\]

where \( p^2(t) = \frac{2W_{pc}^2}{4\pi^2(t)} \), \( P_0 = 2 \cdot 10^{-5} \text{ N/m}^2 \), \( W \) is the emitted sound power, supposed constant, \( W_0 \) is the reference power \( (10^{-12} \text{ W}) \), \( S_0 = 1 \text{ m}^2 \), \( pc \) is the characteristic impedance of the medium, and the factor 2 before \( W \) takes account of the fact of the semi-spherical emission of the source.

We have:

\[
v \cdot dt = dx = \frac{r \cdot d\theta}{\cos \theta} \tag{2}
\]

where \( \theta \) is the angle of view between \( d \) and \( r(t) \), and therefore Eqn. (1) becomes:

\[
L_{eq} = 10 \log \left( \frac{W}{2\pi W_0 T} \int_{0}^{T} \frac{1}{r^2(t)} \cdot \frac{r(t)}{v \cdot \cos \theta} \, dt \right) = L_w + 10 \log \left( \frac{1}{2\pi T} \right) \int_{0}^{\theta} \frac{d\theta}{r(t) \cdot \cos \theta} \tag{3}
\]

with \( \Theta \) being the angle of view of the section travelled in time \( T \).

Also, observing that:

\[
d = r \cdot \cos \theta \tag{4}
\]

from Eqn. (3) we obtain:

\[
L_{eq} = L_w + 10 \log \frac{\Theta}{2\pi Tvd} \tag{5}
\]

where \( L_w = 10 \log \left( \frac{W}{W_0} \right) \).

In first approximation, if \( m \) vehicles pass by in the interval of time \( 2T \), it is necessary to multiply the logarithm argument by \( m \), considering at the same time the mean value of the power of the vehicles:

\[
L_{eq} = L_{wm} + 10 \log m - 10 \log v - 10 \log T_m - 10 \log d + 10 \log \frac{\Theta}{\pi} \tag{6}
\]

where:

\[
m = \text{number of vehicles in } T_m = 2T (s)
\]
\[
v = \text{average speed of the vehicles (m/s)}
\]
\[
\Theta = \text{angle of view to the time } T (\text{rad})
\]
\[
\Theta = \arctan(vT/d), \Theta \rightarrow \pi/2 \text{ for } (vT/d) \rightarrow \infty,
\]

but \([10 \log(\pi/2) - 10 \log(\Theta)] < 0.1 \) for \( vT/d \approx 30 \). In practical situations \( \Theta \) can be assumed to be a constant equal to \( \pi/2 \).
\[
d = \text{distance between the measurement point and the line of traffic flow (m)}
\]
\[
L_{wm} = \text{mean acoustic power of the vehicles (dB(A))}
\]

For every \( i-ma \) category of vehicles (light, heavy, etc.) the sound power level can be calculated by relations of the type:

\[
L_{wi} = a_i + b_i \log(v) \tag{7}
\]

where \( v = \text{average speed of the vehicles} \).

Values for the coefficients \( a_i \) and \( b_i \) are given in the Refs. 8–11.

A comparison between sound power levels calculated from some relations of the type in Eqn. (7), is shown in Ref. 12.

Defining the average power level as:

\[
L_{wm} = 10 \log \left( \sum_{i=1}^{k} \frac{n_i}{m} \cdot 10^{L_{wi}/10} \right) = 10 \log \left( \sum_{i=1}^{k} 10^{L_{wi}/10} p_i \right) \tag{8}
\]

where:

\[
k = \text{number of vehicle categories}
\]
\[
p_{i} = \frac{n_i}{m} = \text{fraction of } i-ma \text{ category vehicles (between 0 and 1)}.
\]

Finally, the \( L_{eq} \) can be expressed as:
3 HYPOTHESIS ON THE STATISTICAL DISTRIBUTION OF TRAFFIC FLOW

Let us suppose that we count the passage of \( m \) vehicles in time \( T_m < 1 \) h. Let us also suppose that we interpret \( m \) as the average value of a symmetrical distribution about \( m \) with lower limit \( \beta m \) and upper limit \( (2-\beta)m \), with \( \beta \) falling between 0 and 1. In general, \( \beta \) can be set according to the information that can be obtained with reference to the change in \( m \).

On an hourly basis it can be estimated that \( N \) vehicles pass, where:

\[
N = 3600m/T_m = km \quad \text{with} \quad k = 3600/T_m
\]

(10)

The analogous distribution for the number of vehicles about \( N \) will therefore have \( \beta km \) as a lower limit and \( (2-\beta)km \) as an upper limit.

3.1 Case of Uniform Distribution for \( m \)

If nothing is known about the distribution of values of \( m \) within the interval of the limits, reference can be made to a uniform distribution in which the probability that the true value falls within the interval is 100% and the values of \( m \) are equiprobable.

For a uniform distribution of limits \( u_+ \) and \( u_- \) the uncertainty \( \Delta u \) is calculated as:

\[
\Delta u = \frac{u_+ - u_-}{2\sqrt{3}}
\]

(11)

For \( m \) it will therefore be:

\[
\Delta m = \frac{m_+ - m_-}{2\sqrt{3}} = \frac{2m - \beta m - \beta m}{2\sqrt{3}} = \frac{m(1 - \beta)}{\sqrt{3}}
\]

(12)

with \( 0 \leq \beta \leq 1 \).

3.2 Case of Triangular Distribution for \( m \)

If it can be considered that the values near the centre of the limits are more likely than values close to the limits, reference can be made to a triangular distribution. In this situation, for a variable \( u \) with limits \( u_+ \) and \( u_- \) the uncertainty \( \Delta u \) is calculated as:

\[
\Delta u = \frac{u_+ - u_-}{2\sqrt{6}}
\]

(13)

Therefore for \( m \) it will be:

\[
\Delta m = \frac{m(1 - \beta)}{\sqrt{6}}
\]

(14)

3.3 Case of Normal Distribution for \( m \)

If it can be considered that the variable \( m \) is distributed according to a normal distribution and that the upper and lower limits define the amplitude interval \( 3\sigma \) including 99.73% of the values, then the uncertainty on \( m \) can be obtained from:

\[
\Delta m = \frac{m(1 - \beta)}{\sqrt{9}}
\]

(15)

3.4 Case of Poisson Distribution for \( m \)

Various authors\textsuperscript{14–17} have referred to this distribution to describe road traffic and this case is therefore of particular interest. In particular it can be considered that the Poisson distribution is applied for flows of less than 600 vehicles/h. The uncertainty on \( m \) for this distribution is given simply by \( \sqrt{m} \).

4 CALCULATION OF UNCERTAINTY ON THE \( L_{EQ} \)

Using the classic theory of errors on Eqn. (9) we have:\textsuperscript{13}

\[
(\Delta L_{eq})^2 = \left( \frac{\partial L_{eq}}{\partial L_{wm}} \Delta L_{wm} \right)^2 + \left( \frac{\partial L_{eq}}{\partial m} \Delta m \right)^2 + \left( \frac{\partial L_{eq}}{\partial v} \Delta v \right)^2 + \left( \frac{\partial L_{eq}}{\partial d} \Delta d \right)^2
\]

(16)

in which:

\[
\frac{\partial L_{eq}}{\partial L_{wm}} = 1;
\[
\frac{\partial L_{eq}}{\partial m} = \frac{c}{\lambda} \quad \text{with} \quad \lambda \quad \text{that represents} \quad m, \quad v, \quad d, \quad \text{and}
\]

\[
\frac{\partial L_{eq}}{\partial d} = \frac{10}{1n(10)}
\]

Recalling Eqn. (8) for \( L_{wm} \), it is possible to obtain the uncertainty \( \Delta L_{wm} \), expressed in terms of uncertainty of the mean\textsuperscript{13}, from:

\[
\Delta L_{wm} = \frac{1}{\sqrt{m}} \sqrt{\sum_{i=1}^{k} 10^{0.1L_{wi} - \psi}} (p_i \Delta L_{wi} + 10 \log(e) \Delta p_i)
\]

(17)

The average power level is always calculated as starting from groups of \( m \) objects: in this sense the uncertainty must be expressed as uncertainty of the mean dividend for \( \Delta m \). In Eqn. (17) \( \Delta L_{wi} \) represents the uncertainty of the single power levels for the \( i-ma \) category of vehicles at a given constant speed.
The uncertainty on $L_{eq}$ can be therefore be expressed as:

$$\frac{\Delta L_{eq}}{\Delta L_{eq}} = \frac{\psi^2}{m} + \left(\frac{\Delta m \cdot c}{m}\right)^2 + \left(\frac{\Delta v \cdot c}{v}\right)^2 + \left(\frac{\Delta d \cdot c}{d}\right)^2$$

(18)

in which, assuming uniform distributions for $v$ and $d$ we have: $\Delta v = \frac{v - v}{2.3}$, $\Delta d = \frac{d - d}{2.5}$ and $\Delta m = \frac{m(1 - \beta)}{n}$ with $n = 3.6$ or $9$ for uniform, triangular or normal distribution. For Poisson distribution, $\Delta m = \sqrt{n}$.

The expressions for the $\Delta L_{eq}$ therefore become:

$$\frac{\Delta L_{eq}}{\Delta L_{eq}} = \frac{\psi^2}{m} + \left(\frac{c(1 - \beta)}{\sqrt{n}}\right)^2 + \left(\frac{c \Delta v}{v}\right)^2 + \left(\frac{c \Delta d}{d}\right)^2$$

(19)

with $n = 3.6$ or $9$ respectively for uniform, triangular or normal distribution and,

$$\frac{\Delta L_{eq}}{\Delta L_{eq}} = \frac{\psi^2 + c^2}{m} + \left(\frac{c \Delta v}{v}\right)^2 + \left(\frac{c \Delta d}{d}\right)^2$$

(20)

in the case of Poisson distribution.

5 DETERMINATION OF THE MEASUREMENT TIME

Inverting Eqns. (19) and (20), keeping the value $\Delta L_{eq}$ fixed and considering Eqn. (10), the value of $T_m$ is obtained according to $N$ (vehicles/h) and $\beta$:

$$T_m = \frac{3600 \cdot \psi^2}{N} \left(\frac{\Delta L_{eq}}{\Delta L_{eq}} - c^2 \left(\frac{1 - \beta}{\sqrt{n}}\right)^2 + \left(\frac{\Delta v}{v}\right)^2 + \left(\frac{\Delta d}{d}\right)^2\right)^{-1}$$

(21)

for uniform, triangular and normal distributions with the same convention for $n$ as before; and

$$T_m = \frac{3600 \cdot (\psi^2 + c^2)}{N} \left(\frac{\Delta L_{eq}}{\Delta L_{eq}} - c^2 \left(\frac{\Delta v}{v}\right)^2 + \left(\frac{\Delta d}{d}\right)^2\right)^{-1}$$

(22)

in the case of Poisson distribution for $m$.

In Eqs. (21) and (22) time $T_m$ must be interpreted as the time necessary to obtain a preset uncertainty on the $L_{eq}$ given a certain number $N$ of vehicles/h associated with a given distribution.

6 RESULTS

By way of example, Fig. 2 shows the measurement times so that the uncertainty on the $L_{eq}$ stays below 2.5 dBA for various values of $\beta$ in the case of uniform distribution for $N$, and with: $L_{w} = 71.9 + 23.8 \log(v)$, $L_{wh} = 84.5 + 18.9 \log(v)$, $\Delta L_{w} = \Delta L_{wh} = 4$ dBA where $l$, $h$=light (weight < 1.5 t) and heavy vehicles (weight $\geq$1.5 t) [Ref. 8]; $v = 13$ m/s; $\Delta v = \Delta d = 0.2$; $p_l = 0.76$; $p_h = 0.24$; $\Delta p_l = \Delta p_h = 0.2$.

Assuming $\beta = 0.5$ and the same data of the previous example, Fig. 3 shows the measurement times so that
the imprecision on the $L_{eq}$ stays below 2.5 dBA for the various distributions considered in relation to the number of vehicles/h.

Obviously, it is possible to use different relations to calculate $L_{wl}$ and $L_{wh}$ other than those previously considered. For example, relations used in the ASJ10 method give sound power levels of vehicles lower than those of Ref. 8 for $v > 13.9$ m/s. However, in the same conditions of the example, the differences between the times calculated are always less than 0.9 minutes (mean of the differences=0.1 minutes). On the other hand, using the relations given in Ref. 9 for $L_{wl}$ and $L_{wh}$ which always give vehicle sound power levels greater than Ref. 8, differences in time measurement of less than 0.2 minutes are obtained for all speeds considered (mean of the differences=0.03 minutes).

Figure 3 also shows a curve obtained from the following reworking of the relation obtained by Fisk5 for the error on the $L_{eq}$ in conditions of low traffic density:

$$T_m = \frac{197136}{N(\Delta L_{eq})^2} \text{ with } T_m \text{ in s.}$$  \hspace{1cm} (24)

Equation (24) takes into account only the hourly traffic flow and identifies significantly shorter times to guarantee the same $\Delta L_{eq}$.

7 CONCLUSIONS

Starting from a simple model for predicting the $L_{eq}$, expressions have been obtained for the error associated with the model and for the minimum measurement time for containing the $\Delta L_{eq}$ within preset values. Through the use of relations that link sound power to the speed of the vehicles, the model allows the various situations of mixed traffic to be evaluated.

However, the model does not take meteorological conditions into consideration. Since the influence of weather increases with the distance, the model can be applied within few tens of meters from the line of traffic where variations in such variables can still be considered very small. Furthermore, variation due to instrument uncertainty has been considered negligible compared to the other principal factors of uncertainty which have been taken into consideration. Variables such as road gradient or road surface, not expressed in the model, can nevertheless be taken into consideration by the use of opportune coefficients in Eqn. (7). In this regard it should be underlined that application of different relations of the type $L_{wl}=a_i+b_i \log(v)$ has not proved to be critical in the calculation of the minimum integration time.

Various statistical hypotheses have been proposed for describing vehicular flow: uniform, triangular, normal and Poisson. Also, a suitable coefficient allows modulation of the limits of the statistical distributions.

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Fig. 3—Variation in minimum measurement time, $T_m$ minutes, assuming uniform, Poisson, triangular and normal distribution in traffic flow, $N$ veh/h, for values of $\beta=0.5$ and $\Delta L_{eq}=2.5$ dBA. The relationship derived by Fisk is included.
considered. The results obtained can also be summarized according to the rate of vehicles per second \( \eta = N/3600 \), respectively as:

\[
T_m = \frac{\eta \beta}{\sqrt{n}} \left\{ (\Delta L_{eq})^2 - c^2 \left[ \left( \frac{1 - \beta}{\sqrt{n}} \right)^2 + \left( \frac{\Delta v}{v} \right)^2 \right] \right\}^{-1}
\]

for uniform, triangular or normal vehicle distributions \((n=3, 6, 9\) respectively), and

\[
T_m = \frac{\eta \beta}{\sqrt{n}} \left\{ (\Delta L_{eq})^2 - c^2 \left[ \left( \frac{\Delta v}{v} \right)^2 + \left( \frac{\Delta d}{d} \right)^2 \right] \right\}^{-1}
\]

for Poisson distribution.

In particular, the relations obtained in this work also allow evaluation of the uncertainties regarding the distance of the observer from the line of traffic and the speed of the vehicles.

The importance of these variables can be quantified by analysing the term on the denominator of Eqn. (26) for which \((\Delta L_{eq})^2 > c^2[(\Delta v)^2 + (\Delta d)^2] \), so leading immediately to \(\Delta L_{eq} > 1.22\) dBA for \(\frac{\Delta v}{v} = \frac{\Delta d}{d} = 0.2\).

Also introducing other parameters related to the distribution of \(m\) (Eqn. (25)) we have \((\Delta L_{eq})^2 > c^2[(\Delta v)^2 + (\Delta d)^2 + (\Delta v/d)^2] \), which for \(\beta = 0.5\), \(n = 3\) (uniform distribution) and \(\frac{\Delta v}{v} = \frac{\Delta d}{d} = 0.2\), gives \(\Delta L_{eq} > 1.75\) dBA.

The calculation carried out by way of example in standard conditions for variables \(v\) and \(d\), shows that it is possible to determine the hourly \(L_{eq}\) with an associated imprecision of \(\Delta L_{eq} = 2.5\) dBA, with measurements lasting less than 15 minutes for hourly traffic flows \((N)\) greater than 60 vehicles/hour. These times are further shortened to values of less than 5 minutes for \(N > 170\) vehicles/hour.

8 REFERENCES